# How to Write Good Problems

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## §1 Introduction

Rather than defining what a good problem is (which is impossible), I'll explain some qualities that all good problems have. Note that these aren't strict rules, but rather recommendations to guide you.

## §2 Basics

You've probably heard all of these.

1. Don't make 'plug and chug' problems (problems that are really just plugging numbers into formulas).
2. Motivated problems (the solution is findable, it's not 'black magic').
3. Every bit of the problem should be used in the solution.

These rules seem simple, but I've seen many problems that do not follow these guidelines.

**Example 2.1** (EGMO 2017/3)
There are 2017 lines in the plane such that no three of them go through the same point. Turbo the snail sits on a point on exactly one of the lines and starts sliding along the lines in the following fashion: she moves on a given line until she reaches an intersection of two lines. At the intersection, she follows her journey on the other line turning left or right, alternating her choice at each intersection point she reaches. She can only change direction at an intersection point. Can there exist a line segment through which she passes in both directions during her journey?

*Proof.* The answer is no.

The main idea is to color the regions formed in the plane black and white in a checkerboard fashion such that no two adjacent regions have the same color (it is well known that this is possible, and not too difficult to prove using induction). Assume WLOG that Turbo starts with a black region to the left. Then, a black region is always to her left no matter what, and we are done! □

This problem seems nice, but it does not satisfy one of the criteria above. In particular, the information that Turbo alternates turning left and right is completely useless, and without this the problem still remains the same. As you can see, 'bad' problems can even slip into very prestigious olympiad contests!

While I don't like this problem very much, there is some justification for the alternation of Turbo's moves. If they gave her the choice in choosing which direction to turn in, her movement has too much freedom and this helps motivate a parity-based solution.

**Example 2.2** (INTEGIRLS 2024 Spring HS Relay 2/1)

A 1 by $n$ bar of chocolate consisting of $n$ 1 by 1 squares of chocolate cannot be split such that any 1 by 1 square of chocolate is broken in half. How many ways are there to split a bar of chocolate with dimensions 1 by 10 with at most 9 splits allowed?

*Proof.* The bar can either be split or not split at every intersection of two points, of which there are 9. Thus, our answer is just $2^9 = 512$. □

This problem is not a bad beginner problem at all, but the last part of the problem statement is completely useless, because it is obvious that more than 9 splits cannot be made. When making a problem statement, you should be careful to make it as concise as possible and avoid repeating unnecessary information like in this problem.

**Example 2.3** (USAMO 2024/5)

Point $D$ is selected inside acute triangle $ABC$ so that $\angle DAC = \angle ACB$ and $\angle BDC = 90^\circ + \angle BAC$. Point $E$ is chosen on ray $BD$ so that $AE = EC$. Let $M$ be the midpoint of $BC$. Show that line $AB$ is tangent to the circumcircle of triangle $BEM$.

*Proof.* My solution to this problem was somewhat longer, but it is pretty much the same as this solution by Khina on AoPS:

Let $AD \cap (ABC) = B'$, and let $F$ be point on $BE$ past $E$ such that $FB = 2BE$.

We claim that $DFCB'$ is cyclic. Note that by definition, $B'F \perp AC$. So we have $\angle FB'C = 90 - \angle ACB' = 90 - \angle BAC = 180 - \angle BDC = \angle FDC$.

We now have that $\angle BFC = 180 - \angle AB'C = 180 - \angle ABC$, implying that $(BFC)$ is tangent to $AB$. Finally, take a homothety with scale factor $\frac{1}{2}$ at $B$, and we are done. □

Motivating the construction of $B'$ and $F$ in this problem is very hard, and this is why this problem is arguably the hardest problem to ever appear on the USAJMO. I eventually stumbled across this idea, but I was not close in-contest. This problem is no doubt very nice, but, again, there's almost no motivation behind its solution.

**Example 2.4** (My friend's geometry test)

Points $A$, $B$, $C$, and $D$ lie on a circle. Let the intersection of lines $AD$ and $BC$ be $X$. It is given that $XA = 7$, $XB = 14$, and $XC = 28$. Find $XD$.

*Proof.* The answer is 56, use Power of a Point. □

You get the idea. 'Plug and chug' problems such as these should not appear on contests (or geometry tests).

Here's an example of an amazing problem that obeys all of the principles above.

**Example 2.5** (Integirls Spring 2023 HS Indiv/27 (Minerva You))

Let $a_1 < a_2 < a_3 < \dots a_{26} < a_{27}$ be the positive factors of 1764. If $\frac{1}{a_1+42} + \frac{1}{a_2+42} + \dots + \frac{1}{a_{26}+42} + \frac{1}{a_{27}+42}$ is equivalent to $\frac{m}{n}$ where $m$ and $n$ are relatively prime positive integers, compute $m \cdot n$.

*Proof.* Note that $1764 = 42^2$. Observe that for any $x$,

$$\frac{1}{x+42} + \frac{1}{\frac{42^2}{x}+42} = \frac{x + \frac{42^2}{x} + 84}{2 \cdot 42^2 + 42\left(x + \frac{42^2}{x}\right)} = \frac{1}{42}.$$

Now, for each factor $n$ of 1764, pair it up with $\frac{1764}{n}$, and sum this up to get a total value of $\frac{13.5}{42} = \frac{9}{28}$. Our answer is 252. □

This proof seems a little bit unmotivated at first, but the idea to pair the factors of 1764 is somewhat common and motivated because there is no other easy way to tackle the sum. This problem is also obviously not a 'plug and chug' problem. I really enjoyed this problem when I first saw it, and I think it's a great example of a good problem that obeys the basic rules stated at the start of this handout.

## §3 Make layered problems

Most good problems have multiple ideas that work together nicely (I say most because this is very difficult to do with easy problems). This generally means that the problem would have multiple steps. There are notable exceptions to this such as USAJMO 2018/6, which has a one-line proof, but is still a great, incredibly hard problem.

Here's a rather infamous example of a problem which isn't very layered at all:

**Example 3.1** (USAJMO 2024/1)

Let $ABCD$ be a cyclic quadrilateral with $AB = 7$ and $CD = 8$. Points $P$ and $Q$ are selected on segment $AB$ such that $AP = BQ = 3$. Points $R$ and $S$ are selected on segment $CD$ such that $CR = DS = 2$. Prove that $PQRS$ is a cyclic quadrilateral.

*Proof.* There are two easy ways to approach this. One way is to observe that both quadrilaterals have the same circumcenter and then use Pythagoras, and the other is to intersect $AB$ and $CD$ and use Power of a Point (though you would have to consider the case where $AB \parallel CD$ here). Both of these ideas work very easily.

I'll present the second way. Observe that if $AB \parallel CD$, $PQRS$ is an isosceles trapezoid so we are done. Otherwise, let $AB \cap CD = E$, and assume WLOG that $E$ is closer to $B$ than $A$ and closer to $C$ than $D$. Let $BE = x$ and $CE = y$. Then, by Power of a Point on $ABCD$, we have that $x(x+7) = y(y+8)$, which implies that $x^2 + 7x = y^2 + 8x$, so $x^2 + 7x + 12 = y^2 + 8x + 12$. However, this means that $(x+3)(x+4) = (y+2)(y+6)$, which implies that $PQRS$ is cyclic. □

This problem is not a great problem at all, and it's really a one-step problem. This means that after the solver figures out the idea of the problem, it becomes an instant solve.

**Example 3.2** (Integirls Spring 2024 MS Team/15 (Scarlet Gitelson))

In trapezoid $ABCD$, with $AB$ parallel to $CD$, $AB = 36$, $CD = 12$, $\angle A = 60^o$, and $\angle B = 30^o$. Let $E$ be the foot of the altitude from $D$ to $AB$, and let $F$ be the intersection of $AC$ and $DE$. Find $AF^2$.

*Proof.* This is a great example of a layered problem. The author's solution resembles a coordinate bash, but I present a more synthetically flavored solution.

Let $G = AD \cap BC$. Then, $ABG$ is a 30-60-90 right triangle. This is also true for triangle $ADE$.

By above, we compute that $AG = 18$ and so $AD = 12$. Hence, $AE = 6$. Additionally, $CD = 12$. By similar triangles/directed lengths, we conclude that $\frac{AF}{CF} = \frac{AE}{CD} = \frac{1}{2}$, and so $\frac{AF}{AC} = \frac{1}{3}$. Finally, by the law of cosines, $AC = \sqrt{3 \cdot 12^2} = 12\sqrt{3}$. Hence, $AF = 4\sqrt{3}$ and our answer is 48. □

The solution to this problem can really be split into two parts: constructing $G$ (which is pretty motivated since it creates 30-60-90 triangles and is a common idea in trapezoid problems), and finishing the argument using similar triangles.

Here is another problem that I proposed which is nice but relies mainly on one idea.

**Example 3.3** (INTEGIRLS Spring 2024 HS Team/12 (Amogh Akella))

Given that $x$, $y$, and $z$ are prime numbers satisfying $x^2 = 40y^2 + z^2$, find the product of all possible values of $z$.

*Proof.* Observe that every prime squared mod 6 is equal to 1, except for 2 and 3. Therefore, $x^2$ must be 1 mod 6. The possible values for $z^2$ are therefore 1 and 3 mod 6.

In the former case, $y$ must be equal to 3 and therefore $x^2 - 360 = z^2$ or $x^2 - z^2 = 360$. Therefore, $(x + z)(x - z) = 360$, and our only solution to this with $x$, $z$ prime is $(23, 13)$. In the latter case, $z$ must be equal to 3, and we obtain a solution of $(13, 2, 3)$. Therefore, the product of all possible values of $z$ is 39. □

The solution for this problem relies mainly on one idea, and that is to take mod 6. The motivation for this is not too difficult to find, the idea is that we find solutions at $y = 2$, 3, so we take mod $2 \cdot 3 = 6$ to eliminate all other cases. However, once this idea is found, the problem is actually quite simple to work out.

The concept of a layered problem is simple enough, but there's one important thing about this that you need to keep in mind.

**Example 3.4** (Answer extraction)

Blah blah blah... The length $XY$ can be written as $2^a 3^b 5^c$ for integers $a$, $b$, $c$. Find $a + b + c$.

The end of this problem is answer extraction, and is quite simple to do. Despite the fact that this part of solving the problem is separate from the rest of the problem, it should not count as another layer to the problem, because it is not just quite simple, but because the author clearly mentioned it in the problem as a step.

Answer extraction in a problem should usually not count as an additional layer unless it is nontrivial to do and actually involves heavy machinery or some insight. Problems like these often use generating functions, the Chinese remainder theorem, or other methods.

A common example of this is answer extractions on the AIME. Contestants are often asked to compute a value mod 1000, and one of the best ways to do this is to compute it mod 8 and mod 125, and then use the Chinese remainder theorem.

Now, I'll give an example of a problem that I made in the past and explain how I came up with the full statement and really developed the problem.

> **Example 3.5**
> Let $ABC$ be a triangle with $\angle BAC = 60°$ and circumcenter $O$. Given that the Euler line of $ABC$ intersects $BC$ at $X$, show that $AX = OX$.

*Proof.* Let $M$ and $N$ be distinct points such that $AMO$ and $ANO$ are equilateral. Additionally, let $H$ be the orthocenter of $ABC$. By an angle chase, $BHOC$ is cyclic. Additionally, it is well known that $AO = AH$ (provable by a simple trigbash). Hence, $MOHN$ is cyclic with center $A$. Now, radical axis on $(ABC)$, $(BHOC)$, and $(MOHN)$ shows that $X$ is on $MN$, which pretty much finishes. □

This problem actually started as one fact, in particular that $BHOC$ is cyclic if $\angle A$ is $60°$. I then observed that $AO = AH$, and proceeded to construct a third circle so that radical axis could work (because I already had two). Finally, after constructing the intersection points of $(MOHN)$ and $(ABC)$, I cleaned up the problem statement by removing $M$, $N$, and $H$ entirely. This problem statement seems very natural, but in fact, it's pretty contrived!

## §4 Copying isn't (necessarily) bad

> **Example 4.1** (Rejected from Integirls Spring 2023 (Amogh Akella))
> Suppose that the value of
> $$\sum_{n=1}^{9}\left((-1)^{n+1}\sum_{m=n}^{9}\cos\frac{n\pi}{2m+1}\right)$$
> can be expressed as $\frac{a}{b}$ such that $a$ and $b$ are relatively prime positive integers. Compute the value of $a+b$.

This problem was rejected purely due to its difficulty. See if you can spot its idea, and see if you can recognize this from a past problem.

> **Example 4.2** (IMO 1963/5)
> Prove that $\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} = \frac{1}{2}$.

*Proof.* Use the 7th roots of unity.

In particular, observe that if $\omega_i$ are the seventh roots of unity, then $\sum_{n=0}^{6}\omega_n = 0$, which means that $\sum_{n=1}^{6}\omega_n = -1$. We really only care about the real part of this, and observe that the real part of this is equal to double the value of $\sum_{n=1}^{3}\omega_n$. Taking the real part, we get $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$, and negating this value finishes the proof.

In general,

$$\sum_{k=1}^{n}(-1)^{k+1}\cos\frac{k}{2n+1}=\frac{1}{2}.$$

□

In fact, the generalized form of this IMO problem is the main idea used in the previous problem. The way to solve the previous problem is to swap the sum and apply this generalized form on the new summands. Each summand is then $\frac{1}{2}$, which gives an answer of $\frac{9}{2} \to 11$.

The previous problem is a very nice extension of this IMO problem, and it's quite hard to see that the problems are connected. There's really nothing wrong with copying the idea of a problem like this, as long as it is concealed and the problem contains some new ideas.

Here is yet another problem that I found which is also related to the aforementioned IMO problem (in fact, this is less well concealed, and I have no idea if the similarity to the IMO problem was intended).

**Example 4.3** (CMIMC 2023 T7)
Compute $\sin^2\frac{\pi}{7}+\sin^2\frac{3\pi}{7}+\sin^2\frac{5\pi}{7}$.

*Proof.* Consider the quantity $3-\cos^2\frac{\pi}{7}-\cos^2\frac{3\pi}{7}-\cos^2\frac{5\pi}{7}$, which is equal to what we would like to find in the problem. Write this as

$$\frac{3}{2}-\frac{1}{2}\left(2\cos^2\frac{\pi}{7}+2\cos^2\frac{3\pi}{7}+2\cos^2\frac{5\pi}{7}-3\right).$$

Observe that $\cos 2\theta = 2\cos^2\theta - 1$, so we can write the above expression as

$$\frac{3}{2}-\frac{1}{2}\left(\cos\frac{2\pi}{7}+\cos\frac{6\pi}{7}+\cos\frac{10\pi}{7}\right).$$

However, this is simply

$$\frac{3}{2}+\frac{1}{2}\left(\cos\frac{\pi}{7}-\cos\frac{2\pi}{7}+\cos\frac{3\pi}{7}\right)=\frac{7}{4}.$$

□

## §5 Theory

This section is related to the previous section, but a little more specific about how much of the problem should be an existing idea.

We define theory as a *somewhat* well-known idea that is either nontrivial to come up with or to prove. For example, the orthocenter is not considered theory because, well, it's incredibly well-known. The triangle inequality is not considered theory because it is quite obvious.

We also define hard theory as theory that is pretty unknown and really hard to come up with or to prove. Most good problems do not involve hard theory, and you should try to omit it while problem writing (there are a couple counterexamples though). Here is an example of a problem requiring hard theory to solve:

**Example 5.1** (USA TSTST 2023/8)

Let $ABC$ be an equilateral triangle with side length 1. Points $A_1$ and $A_2$ are chosen on side $BC$, points $B_1$ and $B_2$ are chosen on side $CA$, and points $C_1$ and $C_2$ are chosen on side $AB$ such that $BA_1 < BA_2$, $CB_1 < CB_2$, and $AC_1 < AC_2$.

Suppose that the three line segments $B_1C_2$, $C_1A_2$, $A_1B_2$ are concurrent, and the perimeters of triangles $AB_2C_1$, $BC_2A_1$, and $CA_2B_1$ are all equal. Find all possible values of this common perimeter.

*Proof.* This is essentially just Brianchon's theorem, stated below. □

**Theorem 5.2**

If a hexagon is circumscribed around a circle, its long diagonals are concurrent.

See if you can finish the proof and find the use of Brianchon's here. Brianchon's theorem is pretty difficult to prove, so this problem does indeed involve some pretty heavy machinery. Indeed, this problem was generally hated, and I don't like it very much either, it isn't a great problem. Here's another example of hard theory, one which I personally consider much worse:

**Example 5.3** (CMIMC 2024 G8)

Let $\omega$ and $\Omega$ be circles of radius 1 and $R > 1$ respectively that are internally tangent at a point $P$. Two tangent lines to $\omega$ are drawn such that they meet $\Omega$ at only three points $A$, $B$, and $C$, none of which are equal to $P$. If triangle $ABC$ has side lengths in a ratio of $3 : 4 : 5$, find the sum of all possible values of $R$.

*Proof.* Use the trigonometric Mixtilinear incircle configuration to obtain an answer of $\frac{11}{2}$. □

Any problem for which its official solution involves something resembling the "trigonometric Mixtilinear incircle configuration" should not be placed on a contest.

While hard theory should be avoided at all costs, theory in general is not bad at all. However, problems that are all theory are not good. This is quite easy to see, because these problems give an advantage to people who know the theory in the problem. Here is one example of a problem that is pretty much all theory:

**Example 5.4** (INTEGIRLS Spring 2024 HS Team/11 (Amogh Akella))

Alex, Jiseop, and Timmy are participating in a coding competition, where they each score an integer number of points between 0 and 400. The sum of the squares of Timmy and Jiseop's scores is equal to the square of Alex's score. Furthermore, each of the three scores is relatively prime. Compute the maximum possible sum of the three scores.

*Proof.* We claim that the maximum sum arises when the scores are 397, 325, and 228, which would mean our answer is 950.

It is well known that every primitive Pythagorean triple can be written as $2rs$, $r^2 - s^2$, and $r^2 + s^2$ for positive integers $r$ and $s$. To maximize the sum, we must maximize $2r^2 + 2rs = 2r(r + s)$.

Additionally, $r < 20$ from the bound. We claim that the maximum possible value of this arises when $r = 19$ and $s = 6$. Clearly when $r$ is some constant, the maximum possible $s$ is $\lfloor \sqrt{400 - r^2} \rfloor$. Additionally, $r(r + s) \le r(r + \sqrt{400 - r^2})$. By taking the derivative or by inspection, this value is clearly less than 475 for all $r < 18$. Additionally, $19(6 + 19) = 475$, so we conclude that $r = 18$, 19, or 20. For $r = 18$, the maximum possible value is $18 \cdot 26 = 468$, and for $r = 20$, the maximum possible value is just $20^2 = 400$. Thus, $(r, s) = (19, 6)$ is optimal and we are done. □

While the rewriting of the Pythagorean triple is pretty well-known, this should still be considered theory, because this is not super well-known, or easy to prove at all. Additionally, this problem is really trivialized by this observation, so this problem is pretty much all theory, and honestly not that great of a problem.

While too much theory is bad, a moderate amount of theory actually makes for nice problems. It's difficult to come up with new problems without existing ideas, but problems that take ideas and build on them can be very nice (this is similar to what I stated in the previous section). Here's an example of a great problem that builds on theory nicely:

**Example 5.5** (USAMO 2023/4)

A positive integer $a$ is selected, and some positive integers are written on a board. Alice and Bob play the following game. On Alice's turn, she must replace some integer $n$ on the board with $n + a$, and on Bob's turn he must replace some even integer $n$ on the board with $n/2$. Alice goes first and they alternate turns. If on his turn Bob has no valid moves, the game ends.

After analyzing the integers on the board, Bob realizes that, regardless of what moves Alice makes, he will be able to force the game to end eventually. Show that, in fact, for this value of $a$ and these integers on the board, the game is guaranteed to end regardless of Alice's or Bob's moves.

*Proof.* Use $\nu_2$. If $\nu_2(n) < \nu_2(a)$, it is not difficult to show that the game must end no matter what moves are played. Otherwise, it is not that difficult to show that the game will never end. □

While this problem uses prime exponents ($\nu_p$), it builds on it very nicely, even disguising the number-theoretic idea in a combinatorial statement. This problem is widely considered way too easy for its positioning on the USAMO and the USAJMO, but it is still quite a nice problem, and quite nice to work out.

## §6 Answer Extraction

This section is really just my two cents about answer extractions in problem statements.

I've seen many problems that ask for things like $a + b + c$ given that some desired value is $\frac{a\sqrt{b}}{c}$. I generally don't like these kinds of problems. It feels like the problem is asking for something unnatural.

**Example 6.1** (Amogh Akella)
Let $ABCD$ be a trapezoid with $AD = 2\sqrt{3}$, $BC = 3$, and $\angle BAD = \angle ADC = 30^\circ$. Compute the perimeter of $ABCD$.

*Proof.* Let $E$ be the foot from $B$ to $AD$, and $F$ the foot from $C$ to $AD$. Observe that $AE = DF = \sqrt{3} - \frac{3}{2}$, and so $AB = CD = \frac{2}{\sqrt{3}}\left(\sqrt{3} - \frac{3}{2}\right) = 2 - \sqrt{3}$. Therefore, our answer is $3 + 2\sqrt{3} + 2 - \sqrt{3} + 2 - \sqrt{3} = 7$. □

Notice that this problem asks for a much more natural value, the perimeter of the said trapezoid.

You might notice that the value of the perimeter of the trapezoid actually magically turns out to be an integer, but this is not by accident; it's by construction. I'll briefly describe how I came to the specific side lengths below.

Let the difference between the bases be $x$. Then, $AE + AB + CD + DF = x + \frac{2x}{\sqrt{3}} = \frac{1}{3}x\left(3 + 2\sqrt{3}\right)$. The key here is that if we set $x = 2\sqrt{3} - 3$, the radicals cancel well and we are left with $\frac{1}{3}(12 - 9) = 1$. Hence, if the difference between the top base and the bottom base is $2\sqrt{3} - 3$, our final answer will be $2BC + 1$. Finally, if we let $BC = 3$, notice that our answer does not just turn out to be an integer, the value of our other base is also $2\sqrt{3}$, which is not a very ugly radical!

I'd like to briefly note that while this problem is nice, it's not ready to put on a contest yet because the value of the answer, 7, is very guessable. In fact, a sufficiently nicely drawn diagram should make it obvious that the answer is 7. One way that this can be dealt with nicely is scaling the sides up by an integer amount.

**Example 6.2** (AIME II 2024/4)
Let $x, y$ and $z$ be positive real numbers that satisfy the following system of equations:

$$\log_2\left(\frac{x}{yz}\right) = \frac{1}{2}$$

$$\log_2\left(\frac{y}{xz}\right) = \frac{1}{3}$$

$$\log_2\left(\frac{z}{xy}\right) = \frac{1}{4}$$

Then the value of $\left|\log_2(x^4y^3z^2)\right|$ is $\frac{m}{n}$ where $m$ and $n$ are relatively prime positive integers. Find $m + n$.

*Proof.* I was too lazy to actually write this up so here is akliu's writeup on the AoPS wiki:

Denote $\log_2(x) = a$, $\log_2(y) = b$, and $\log_2(z) = c$.

Then, we have $a - b - c = \frac{1}{2}$, $-a + b - c = \frac{1}{3}$, and $-a - b + c = \frac{1}{4}$.

Now, we can solve to get $a = \frac{-7}{24}, b = \frac{-9}{24}, c = \frac{-5}{12}$. Plugging these values in, we obtain $|4a + 3b + 2c| = \frac{25}{8} \implies \boxed{033}$. □

In my opinion, this problem is just ordinary, and the answer extraction is terrible. There's no real clever way to solve this problem other than convert it to a system of equations, and the answer extraction is completely unnecessary. $x^4y^3z^2$ is a seemingly arbitrary expression, and it doesn't even yield an integer value.

In my opinion, the constants in a problem should be chosen so that the final answer is always nice, and the answer extraction isn't just weird. I've seen many problems in contests disobey this rule, but I always find it nice when a problem magically turns out to have an integer answer.

Another advantage of having answers magically simplify is that it helps contestants not make sillies. A problem on a contest should be a test of a contestant's problem-solving ability rather than their ability to not make sillies. With problems which have nice answers that simplify well, contestants will know that they made a silly if they obtain an answer that is not nice and does not fit the answer format.

## §7 Make the problem flow

This section isn't directly related to the mathematical content of problems, but it's still equally (if not more) important than the other sections. If a problem is really badly worded, or it generally looks scary, it should be rephrased.

### §7.1 Don't make scary problem statements.

Most people don't have scaring people away as their primary objective of writing problems. Your problems should not be scary to anyone who is reading them. Here are a couple of examples of things that are potentially scary and shouldn't appear on contests:

- Big blocks of text. As an English antimain, I can confirm that these are scary.
- Large mathematical expressions.

**Example 7.1** (USAMO 2024/6)

Let $n > 2$ be an integer and let $\ell \in \{1, 2, \dots, n\}$. A collection $A_1, \dots, A_k$ of (not necessarily distinct) subsets of $\{1, 2, \dots, n\}$ is called $\ell$-large if $|A_i| \geq \ell$ for all $1 \leq i \leq k$. Find, in terms of $n$ and $\ell$, the largest real number $c$ such that the inequality

$$\sum_{i=1}^{k}\sum_{j=1}^{k} x_i x_j \frac{|A_i \cap A_j|^2}{|A_i| \cdot |A_j|} \geq c \left(\sum_{i=1}^{k} x_i\right)^2$$

holds for all positive integers $k$, all nonnegative real numbers $x_1, \dots, x_k$, and all $\ell$-large collections $A_1, \dots, A_k$ of subsets of $\{1, 2, \dots, n\}$. Note: For a finite set $S$, $|S|$ denotes the number of elements in $S$.

Seriously, this is scary. Despite being "only" 40 MOHS, this problem received one solve in contest, which was by a pretty famous USA IMO team member.

### §7.2 Don't make clunky problem statements.

We've all seen clunky problem statements, though they don't appear in top-level contests often because the problem writers are experienced and know how to make their problem statements really *flow*.

It may be effortless for top-level problem writers, but you should always pay attention to making sure your problem statements don't sound clunky!

## §8 Don't make troll problems.

Seriously, don't.

## §9 Acknowledgements

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